Pattern Recognition
as a
Human Centered non-Euclidean Problem

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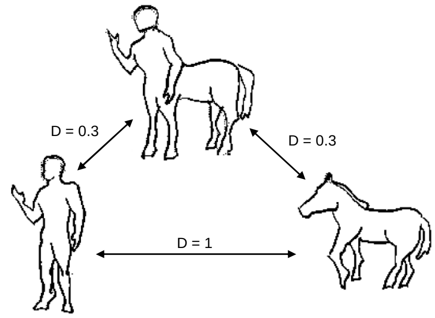
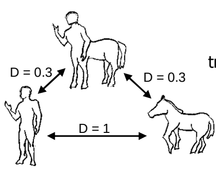
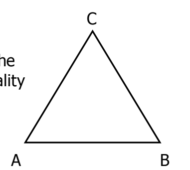


David W. Jacobs, Daphna Weinshall and Yoram Gdalyahu, Classification with Nonmetric Distances: Image Retrieval and Class Representation, *IEEE Trans. Pattern Anal. Mach. Intell*, 22(6), pp. 583-600, 2000.

Human Judgments …



violation of the
triangle inequality



D(A,B) < D(A,C) + D(C,B)

… often don't fit in an Euclidean world.

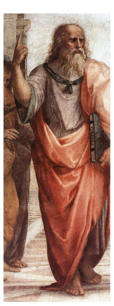
Learning about the world

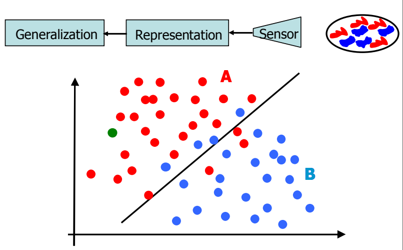
Human knowledge grows
in the debate between

-those who see the patterns, and

-those who know the universal laws

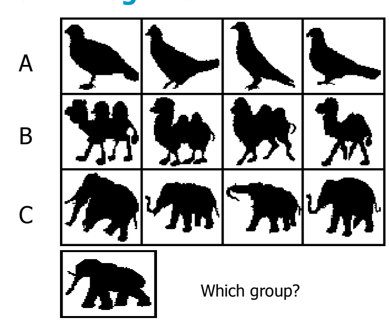
Automatic Pattern Recognition





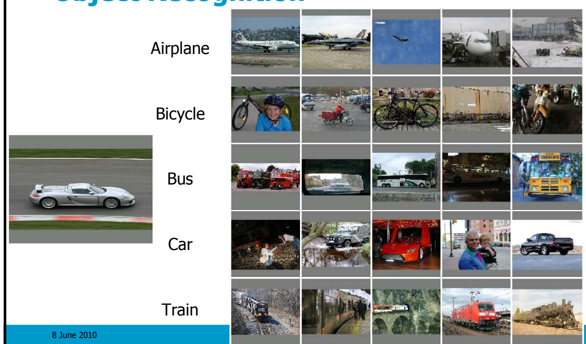
Can we replace our recognition of real world objects by a formal system, also when our mental system is non-Euclidean? How to train? How to apply?

Blob Recognition

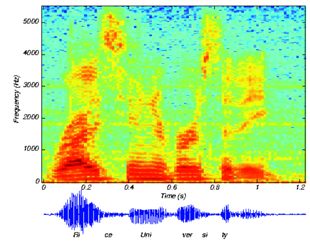


Which group?

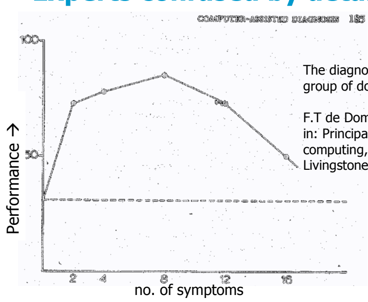
Object Recognition



Pattern Recognition: Speech



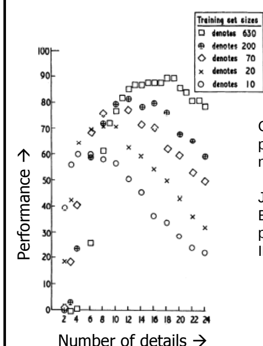
Experts confused by details



The diagnostic classification accuracy of a group of doctors. Sample size is 100.

F.T de Dombal, Computer-assisted diagnosis, in: Principals and practice of medical computing, Whitby and Lutz (eds.), Churchill Livingstone, London, 1971, pp 179 - 199.

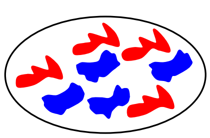
Computers confused by details



Character recognition classification performance as a function of the number of n-tuples used.

J.R. Ullmann,
Experiments with the n-tuple method of pattern recognition,
IEEE Trans. on Computers, 1969, 1135-1136

Pattern Recognition: Shapes

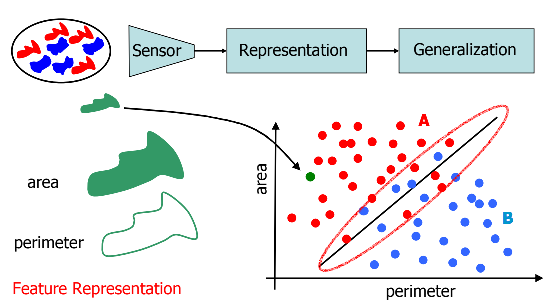


Examples of objects for different classes

Object of unknown class to be classified

A ? B

Pattern Recognition System



Measuring Human Relevant Information

A

B

Nearest neighbours sorted:

B A A A B A A B A B

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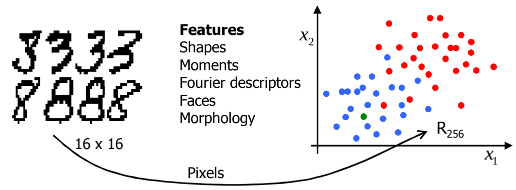
Pixel Representation



Pixels are more general, initially complete representation
Large datasets are available → good results for OCR

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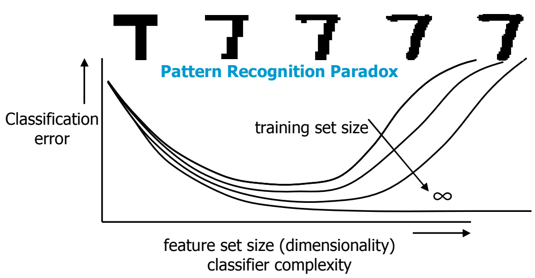
Peaking Phenomenon, Overtraining
Curse of Dimensionality, Rao's Paradox



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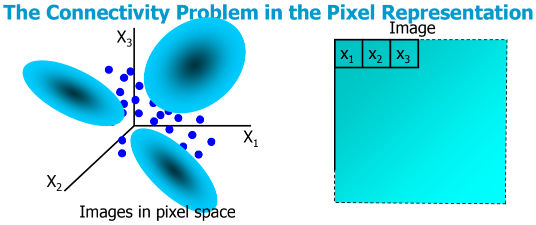
The Connectivity Problem in the Pixel Representation



Dependent (connected) measurements are represented independently.
The dependency has to be refound from the data.

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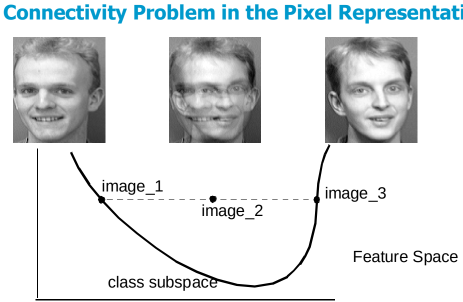
The Connectivity Problem in the Pixel Representation



Interpolation does not yield valid objects

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Representations

Features – details lost

Pixels – no connectivity

Dissimilarities – shape dependent

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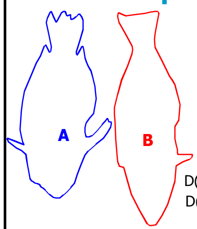
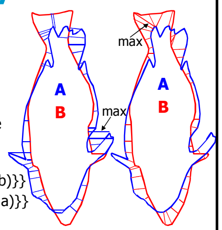
Examples Dissimilarity Measures



Dist(**A**,**B**):
a ∈ **A**, points of **A**
b ∈ **B**, points of **B**
d(a,b): Euclidean distance



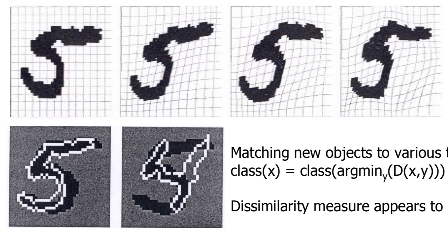
D(**A**,**B**) = max_a{min_b{d(a,b)}}
D(**B**,**A**) = max_b{min_a{d(b,a)}}

Hausdorff Distance (metric):
DH = max{max_a{min_b{d(a,b)}} , max_b{min_a{d(b,a)}}}

D(**A**,**B**) ≠ D(**B**,**A**)

Modified Hausdorff Distance (non-metric):
DM = max{mean_a{min_b{d(a,b)}},mean_b{min_a{d(b,a)}}}

Examples Dissimilarity Measures



Matching new objects to various templates:
class(x) = class(argmin$_y$(D(x,y)))

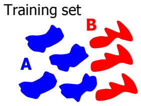
Dissimilarity measure appears to be non-metric.

A.K. Jain, D. Zongker, Representation and recognition of handwritten digit using deformable templates, IEEE-PAMI, vol. 19, no. 12, 1997, 1386-1391.

Dissimilarity Representation

Not used by NN Rule

Training set



Dissimilarities d_{ij} between all training objects

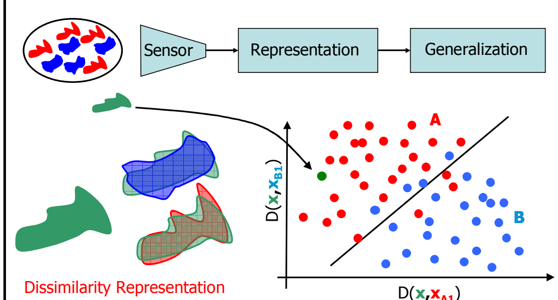
$$D_T = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} & d_{17} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} & d_{27} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} & d_{37} \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{45} & d_{46} & d_{47} \\ d_{51} & d_{52} & d_{53} & d_{54} & d_{55} & d_{56} & d_{57} \\ d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & d_{66} & d_{67} \\ d_{71} & d_{72} & d_{73} & d_{74} & d_{75} & d_{76} & d_{77} \end{pmatrix}$$

Unlabeled object **x** to be classified

$$d_x = (d_{x1}\ d_{x2}\ d_{x3}\ d_{x4}\ d_{x5}\ d_{x6}\ d_{x7})$$

The traditional Nearest Neighbor rule (template matching) finds:
label(argmin$_{trainset}$\{d_{xi}\}) ,
without using D_T. Can we do any better?

Pattern Recognition System



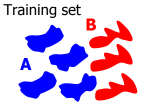
Dissimilarities – Possible Assumptions

Metric

1. Positivity: $d_{ij} \geq 0$
2. Reflexivity: $d_{ii} = 0$
3. Definiteness: $d_{ij} = 0$ objects i and j are identical
4. Symmetry: $d_{ij} = d_{ji}$
5. Triangle inequality: $d_{ij} < d_{ik} + d_{kj}$
6. Compactness: if the objects i and j are very similar then $d_{ij} < \delta$.
7. True representation: if $d_{ij} < \delta$ then the objects i and j are very similar.
8. Continuity of d.

Alternatives for the Nearest Neighbor Rule

Training set



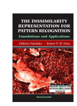
Dissimilarities d_{ij} between all training objects

$$D_T = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} & d_{17} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} & d_{27} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} & d_{37} \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{45} & d_{46} & d_{47} \\ d_{51} & d_{52} & d_{53} & d_{54} & d_{55} & d_{56} & d_{57} \\ d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & d_{66} & d_{67} \\ d_{71} & d_{72} & d_{73} & d_{74} & d_{75} & d_{76} & d_{77} \end{pmatrix}$$

Unlabeled object **x** to be classified

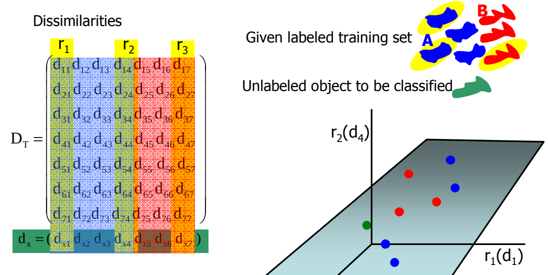
$$d_x = (d_{x1}\ d_{x2}\ d_{x3}\ d_{x4}\ d_{x5}\ d_{x6}\ d_{x7})$$

1. Dissimilarity Space
2. Embedding



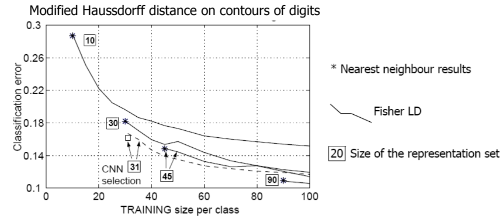
Pekalska, The dissimilarity representation for PR. World Scientific, 2005.

Alternative 1: Dissimilarity Space



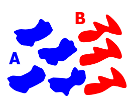
Selection of 3 objects for representation

Dissimilarity Space Classification ←→ Nearest Neighbor Rule



Dissimilarity based classification outperforms the nearest neighbor rule.

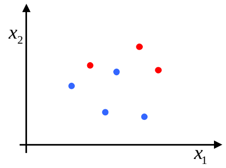
Embedding



Training set

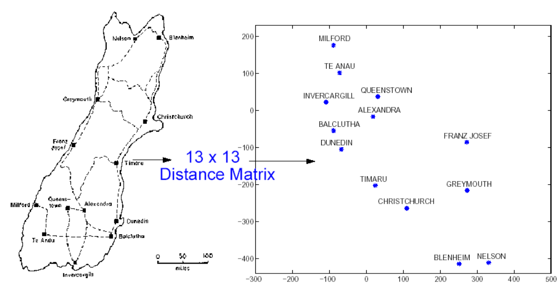
→ Dissimilarity matrix D → X

Is there a feature space for which Dist(X,X) = D ?

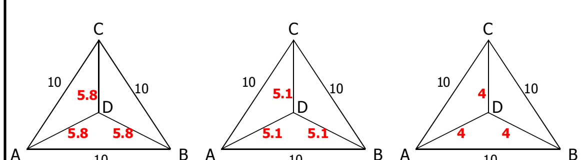


Position points in a vector space such that their Euclidean distances → D

Embedding

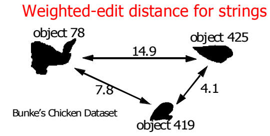


Euclidean - Non Euclidean - Non Metric

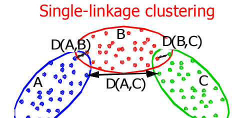


Non-metric distances

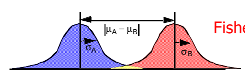
Weighted-edit distance for strings



Single-linkage clustering

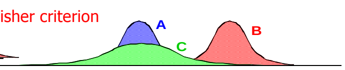


D(A,C) > D(A,B) + D(B,C)



$$J(A,B) = \frac{|\mu_A - \mu_B|^2}{\sigma_A^2 + \sigma_B^2}$$

Fisher criterion



$$J(A,C) = 0 \quad J(A,B) = \text{large}$$

$$J(C,B) = \text{small} \neq J(A,B)$$

(Pseudo-)Euclidean Embedding

$m \times m$ D is a given, imperfect dissimilarity matrix of training objects.

Construct inner-product matrix: $B = -\frac{1}{2} J D^{(2)} J \quad J = I - \frac{1}{m}\mathbf{11}$

Eigenvalue Decomposition , $B = Q \Lambda Q^T$

Select k eigenvectors: $X = Q_k \Lambda_k^{\frac{1}{2}}$ (problem: $\Lambda_k < 0$)

Let $\Im_k$ be a $k \times k$ diag. matrix, $\Im_k(i,i)$ = sign($\Lambda_k(i,i)$)

$\Lambda_k(i,i) < 0 \rightarrow$ Pseudo-Euclidean

$n \times m$ D_z is the dissimilarity matrix between new objects and the training set.

The inner-product matrix: $B_z = -\frac{1}{2}(D_z^{(2)} J - \frac{1}{n}\mathbf{11}^T D^{(2)} J)$

The embedded objects: $Z = B_z Q_k |\Lambda_k|^{-\frac{1}{2}} \Im_k$

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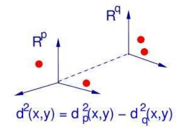
PES: Pseudo-Euclidean Space (Krein Space)

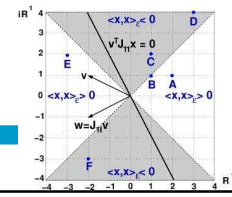
If D is non-Euclidean, B has p positive and q negative eigenvalues.

A pseudo-Euclidean space $\mathcal{E}$ with signature (p,q), $k = p+q$, is a non-degenerate inner product space $\Re_k = \Re_p \oplus \Re_q$ such that:

$$\langle x, y \rangle_\varepsilon = x^T \Im_{pq} y = \sum_{i=1}^{p} x_i y_i - \sum_{j=p+1}^{q} x_j y_j \qquad \Im_{pq} = \begin{bmatrix} I_{p\times p} & 0 \\ 0 & -I_{q\times q} \end{bmatrix}$$

$$d_\varepsilon^2(x,y) = \langle x-y, x-y \rangle_\varepsilon = d_p^2(x,y) - d_q^2(x,y)$$





Pseudo Euclidean Space

Euclidean embedding D → X

$$d_{ij}^2 = \|\mathbf{x}_i - \mathbf{x}_j\|^2$$

Pseudo Euclidean embedding D → {X^p,X^q}

$$d_{ij}^2 = \|\mathbf{x}_i^p - \mathbf{x}_j^p\|^2 - \|\mathbf{x}_i^q - \mathbf{x}_j^q\|^2$$

'Positive' and 'negative' space,

Compare Minkowsky space in relativity theory

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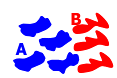
Dissimilarity based classification procedured compared

Training set



→ Dissimilarity matrix D

Test object x

→ Dissimilarities d_x with training set

1. Nearest Neighbour Rule
2. Reduce training set to representation set

⇒ dissimilarity space

3. Embedding:Select large $\Lambda_{ii} > 0$ ⇒ Euclidean space

Select large $|\Lambda_{ii}| > 0$ → pseudo-Euclidean space

} discriminant function

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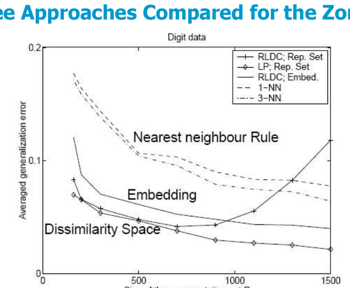
Three Approaches Compared for the Zongker Data



Dissimilarity Space equivalent to Embedding better than Nearest Neighbour Rule

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Non-Euclidean Representations

- Why do we have them?
- Are they essential?
- Can we build classifiers for them? (to some extend)
- Can we transform them into Euclidean representations? (Yes, but at the cost of performance loss)

SIMBAD **Beyond Features**
Similarity-based Pattern Analysis and Recognition

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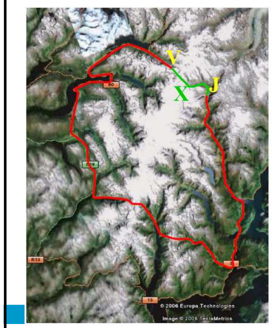
Computational Noise

Even for Euclidean distance matrices zero eigenvalues may show negative, e.g:

- X = N(50,20) : 50 points in 20 dimensions
- D = Dist(X): 50 x 50 distance matrix
- Expected: 49-20 = 29 zero eigenvalues
- Found: 15 negative eigenvalues

Lack of information

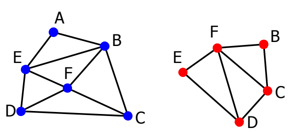


1800:
Crossing the Jostedalsbreen was impossible.
Travelling around (200 km) lasted 5 days.
Untill the shared point X was found.
People could visit each other in 8 hours.

D(V,J) = 5 days
D(V,X) = 4 hours
D(X,J) = 4 hours

Graph Matching → Dissimilarities

Representation by Connected Graphs



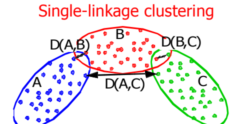
Graph (Nodes, Edges, Attributes)

Distance (Graph_1, Graph_2)

Intrinsicly Non-Euclidean Dissimilarity Measures
Single Linkage

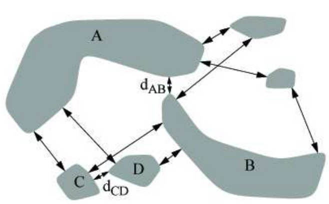
Distance(Table,Book) = 0
Distance(Table,Cup) = 0
Distance(Book,Cup) = 1

Single-linkage clustering



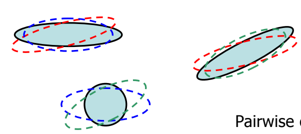
D(A,C) > D(A,B) + D(B,C)

Boundary distances

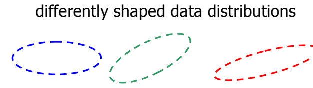


A set of boundary distances may characterize sets of datapoints:
Distances → features

Intrinsicly Non-Euclidean Dissimilarity Measures
Mahalanobis

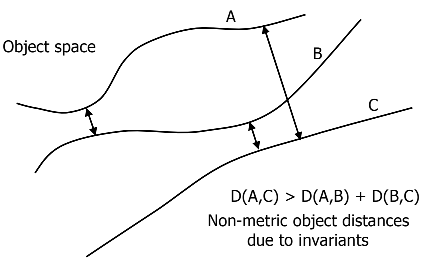


Pairwise comparison between
differently shaped data distributions

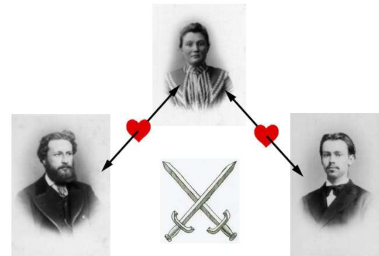


Different pairs → different comparison frameworks
→ non-Euclidean

Intrinsicly Non-Euclidean Dissimilarity Measures
Invariants



Intrinsicly Non-Euclidean Dissimilarity Measures



Non-Euclidean human relations

Objects may have an 'inner life'

In dissimilarity measures the 'inner life' of objects may be taken into account (e.g. invariants).

→ Objects cannot be represented anymore as points

→ Non-Euclidean dissimilarities

Conclusions

- Pseudo Euclidean Space (PES) is sometimes informative (corrections are not helpful).
- The corresponding problems may be intrinsic non-Euclidean
- Classifiers for non-Euclidean data have to be studied further